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A WIND SHEAR MECHANISM FOR  
PRODUCING SPORADIC E  
BY CONCENTRATING MINOR  
METEORIC IONS

BY

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A WIND SHEAR MECHANISM FOR PRODUCING SPORADIC E  
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(Received )

ABSTRACT

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The theory for the development of a Sporadic E layer by the wind shear mechanism operating on slowly recombining minor ions, such as  $\text{Ca}^+$ ,  $\text{Mg}^+$  and  $\text{Na}^+$  is developed. It is shown that if these ions constitute normally about one percent of the total ion abundance in the E region their density at wind shear nodes can rival that of the major ions. This is a consequence of their low effective removal rate. It is suggested that fluctuations in the influx of meteoric material may enable the density of minor ions at E-W wind shear nodes occasionally to dominate the density of major ions thus giving rise to observable Sporadic E.

Author

## 1. INTRODUCTION

In the E region and lower F region the major ions ( $O^+$ ,  $N_2^+$  and  $O_2^+$ ) created by solar radiation and by fast secondary electrons are transformed by ion molecule reactions predominantly to  $O_2^+$  and  $NO^+$ . These diatomic ions then recombine dissociatively with electrons. Except at the lowest fringe of the E region near 85 km the balance between ion production and removal rates requires recombination coefficients for  $O_2^+$  and  $NO^+$  which agree very well with rate coefficients measured in the laboratory<sup>(1, 2)</sup>. This is true at 300°K and at other temperatures also in the case of  $N_2^+$  and  $NO^+$  for which laboratory measurements of temperature dependence have been carried out. At 300° the coefficients are about  $5 \times 10^{-7}$  cm<sup>3</sup>/sec for  $NO^+$ <sup>(1, 3, 4)</sup> and about  $2 \times 10^{-7}$  cm<sup>3</sup>/sec for  $O_2^+$ <sup>(1, 5)</sup>. The dependence on electron temperature, in the neighborhood of  $T_e^{-1.4}$  for  $NO^+$ , agrees with ionospheric data as well as laboratory measurements<sup>(1, 2, 3)</sup>. Furthermore there appears to be no difficulty in accounting for the D region layer near 80 km on the model of nitric oxide ionization by solar Lyman  $\alpha$  and dissociative recombination at a rate of  $1-2 \times 10^{-6}$  cm<sup>3</sup>/sec (in agreement with the  $T_e^{-1.4}$  temperature dependence) if the high NO densities in the  $10^7$  cm<sup>-3</sup> range as measured by Barth<sup>(6)</sup> prevail there<sup>(2)</sup>.

Nevertheless, this seemingly happy concordance has not as yet induced general acceptance of large dissociative recombination coefficients. A most serious deterrent is the difficulty of reconciling the theory of

the wind-shear model of Sporadic-E ionization with recombination coefficients very much greater than  $10^{-8} \text{ cm}^3/\text{sec}$  (7, 8). Since the wind shear mechanism is by far the most attractive one proposed for Sporadic-E and since, furthermore, it is supported by fairly clear cut observational evidence that there is a relationship between the Sporadic-E layers and nodes in the E-W horizontal wind profile (9, 10) it seems desirable somehow to reconcile this theory to effective recombination coefficients of the order of  $3-5 \times 10^{-7} \text{ cm}^3/\text{sec}$  for the major diatomic ions. It is the purpose of this paper to attempt this reconciliation in retaining the wind shear mechanism but in permitting it to operate on minor, slowly recombining metallic ions, such as  $\text{Ca}^+$ ,  $\text{Mg}^+$ , and  $\text{Na}^+$ , presumably meteoric in origin. The central idea here is that the vertical electric fields resulting from vertical gradients in the horizontal wind concentrate all kinds of ions including the minor ones which in overall abundance near 100 km may constitute no more than 1% of all ionic species. The major ions,  $\text{O}_2^+$  and  $\text{NO}^+$ , recombine so swiftly that at the proper nodes in the field they do not rise in density much above their normal space average values. The metal ions, however, should have removal rates which are very low and which depend very probably linearly on the density of the individual species. In fact the rate controlling process probably will be an ion molecule reaction resulting in the formation of a metal oxide ion or a mutual neutralization process. In this case, given the typical preponderance of the concentrating effect of the electric fields over the dispersing effect of diffusion, the build up in ion density can be dramatic and the metallic ion concentrations can reach peaks a hundred times higher than their average values. It is suggested that peaks in

metallic ion concentration always exist at the level where the wind shifts from east to west but that normally these lie below the general level of the  $O^+$ ,  $NO^+$  and  $O_2^+$  ionosphere. On occasions, when there is an enhancement in the amount of meteoric debris present in the upper atmosphere, the peaks can push above the normal ionospheric background and manifest themselves as Sporadic-E layers. Some support for this model exists in that the seasonal variation in frequency of Sporadic-E at temperate latitudes<sup>(11)</sup> greatly resembles the variation in radar meteor rates<sup>(12)</sup>. It is also interesting to note isolated coincidences such as one brought out by Dubin<sup>(13)</sup> in which an intense occurrence of Sporadic-E accompanied a massive micrometeoritic shower in February, 1958. Of course, speculation about a relationship between meteors and Sporadic-E is by no means unknown<sup>(14)</sup>. What is being proposed here is a merger of the meteoric hypotheses and the wind shear mechanism.

## 2. GENERAL WIND SHEAR THEORY

As Whitehead showed in his original treatment of the wind shear theory<sup>(7, 8)</sup> and as others<sup>(15, 16)</sup> have verified in elaborations on his ideas a concentration of ions can be produced by the vertical electric field resulting from the horizontal motion of the atmosphere dragging the ionosphere across the magnetic field of the earth. If there exists a vertical gradient in the E-W component of the wind velocity then the interplay of motional fields, electric fields and diffusion will result in a vertical motion of the ions. If it is assumed that the E wind velocity component varies sinusoidally with the altitude  $z$  then - still according to Whitehead - there will be a vertical velocity of ionized material

$$V_z = -V_o \sin 2\pi z/\lambda \quad (1)$$

where  $\lambda$  is the vertical "wave-length" of the movement.  $\lambda$  usually lies between 5 and 20 km and  $V_0$  between 3 and 25 m/sec. If, now, a given ionic species is created at a rate

$$Q = N_0^+ / \tau_0, \quad (2)$$

where  $N_0^+$  would be the steady state density without the vertical motion, and  $\tau_0$  is the lifetime of the ion against removal either by recombination or by some other sort of collision, then the rate of increase in density of ionic species at some level  $z$  would be given by

$$\frac{\partial N^+}{\partial t} = \frac{N_0^+}{\tau_0} - \frac{N^+}{\tau} + D \frac{\partial^2 N^+}{\partial z^2} + \frac{\partial}{\partial z} N^+ V_0 \sin \frac{2\pi z}{\lambda}. \quad (3)$$

$D$  is, of course, the appropriate Ionic diffusion coefficient.

If a new vertical scale  $\chi$ , given by

$$\chi = 2\pi z / \lambda \quad (4)$$

is adopted and the ratio  $N^+/N_0^+$  is written as  $f(x)$  then in the steady state  $f$  is given by

$$L \frac{d^2 f}{d\chi^2} + R \frac{d}{d\chi} f \sin \chi - f \frac{\tau_0}{\tau} + 1 = 0 \quad (5)$$

where

$$L = 4\pi^2 D \tau_0 / \lambda^2 \quad (6)$$



and

$$R = 2\pi V_0 \tau_0 / \lambda . \quad (7)$$

The boundary conditions are

$$\int_0^{2\pi} f(x) dx = 2\pi \quad (8)$$

and  $f(x) \geq 0$  everywhere.

### 3. THE MAJOR IONOSPHERIC IONS AS PRINCIPAL IONS IN SPORADIC E

Now in the present Sporadic-E theory a single component ionosphere is assumed where  $N^+$  implicitly represents the density of the major ionic species and it is assumed specifically that

$$N^+ \cong N_e \quad (9)$$

and that

$$\tau^{-1} = \alpha N_e \cong \alpha N^+ \quad (10)$$

where  $\alpha$  is the effective recombination coefficient in the E region<sup>(7, 8)</sup>.

The result is that the characteristic build up factor  $R/L$  must attain values in the neighborhood of 5 and this it cannot do unless  $\tau^{-1}$  is sufficiently long. The requirement in fact is that  $\alpha$  be no more than a few times  $10^{-8}$  cm<sup>3</sup>/sec. Under these conditions the steady state equation

involves  $f$  quadratically

$$L \frac{d^2 f}{dx^2} + R \frac{d}{dx} f \sin x - f^2 + 1 = 0 \quad (11)$$

and

$$R = 2\pi V_0 / \lambda \alpha N_0^+ \quad (12)$$

while

$$L = 4\pi^2 D / \lambda^2 \alpha N_0^+. \quad (13)$$

It is the dependence on  $f^2$  which prevents  $f$  from attaining large values at maximum. For example, neglecting diffusion, Whitehead (7, 8) showed that

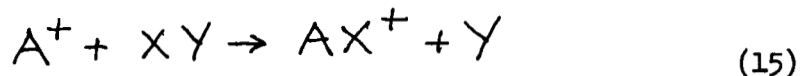
$$f_{Max} = (R + \sqrt{R^2 + 4})/2 \cong R, \quad R \gg 1 \quad (14)$$

and that  $f$  could be of the order of 10 only if  $\alpha$  were of the order of  $10^{-8} \text{ cm}^3/\text{sec}$  when typical values of  $V_0$  and  $\lambda$  are used in determining  $R$ . A very much more efficient concentrating mechanism would be needed to build up a narrow ten fold enhancement in the major atmospheric ions if indeed their recombination coefficients are larger than  $10^{-7} \text{ cm}^3/\text{sec}$  in view of the fact that the disappearance rate in this model increases with the ion density.

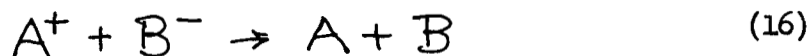
#### 4. WIND SHEAR THEORY FOR MINOR SLOWLY RECOMBINING IONS

If it is allowed that  $\alpha$  is large for these major ions the wind shear mechanism can still be retained provided there exists in the ionosphere an adequate reservoir of very slowly recombining ions whose average abundance may be very low. The ions with the longest lifetime will always tend to be preponderant in the concentration peaks and if the ratio of their lifetime to their average concentration is high enough their density can build up from a minimum far below the density of the surrounding major ions to a sharp maximum which dominates the local distribution.

Two models of the removal process affecting the minor ionic constituents will be considered. One of these is a reaction such as



or



in which

$$\tau^{-1} = k XY \text{ or } k B^- \quad (17)$$

independent of the density of  $A^+$ . The other is recombination with electrons with

$$\tau^{-1} = \alpha N_e \quad (18)$$

where, however, it is not necessarily true that  $N_e$  and  $N^+$  are equal or nearly equal everywhere, the average value of  $N_e$  being under the control of  $O^+$ ,  $O_2^+$  and  $NO^+$ .

#### 4.1 STEADY STATE: ION MOLECULE REACTION REMOVAL

In the first model  $\tau_0 = \tau$  so that Eq. (5) reads

$$\frac{d^2 f}{dx^2} + \frac{R}{L} \frac{d}{dx} f \sin x - \frac{f}{L} + \frac{1}{L} = 0 \quad (19)$$

with

$$R/L = V_0 \lambda / 2\pi D \quad (20)$$

It will be assumed that  $D$  and  $\tau$  are independent of altitude. The ratio of  $R/L$  to  $1/L$  which controls the nature of the problem is then determined by the relative importance of the ionic decay time  $\tau$ . There are two limiting cases. One of these is when  $\tau$  is small enough that  $R/L$  is negligible compared to  $1/L$ . In this case

$$L \frac{d^2 f}{dx^2} = f - 1 \quad (21)$$

and

$$f = 1 \quad (22)$$

is the trivial solution. The other case - when  $\tau$  is very long - amounts to that in which a reservoir of ions which are neither being created nor

destroyed is rearranged in space by the electric field gradient and diffusion. The steady state in this approximation is

$$\frac{d^2 f}{dx^2} + \frac{R}{L} \frac{d}{dx} f \sin x = 0 \quad (23)$$

$$\int_0^{2\pi} f(x) dx = 2\pi \quad (24)$$

$$f(x) \geq 0 \quad (25)$$

One quadrature yields

$$\frac{df}{dx} + \frac{R}{L} f \sin x = \text{const}, \quad (26)$$

For the solution which has a maximum at  $x = 0$  the constant vanishes and

$$f = f_0 \exp(-R(1 - \cos x)/L) \quad (27)$$

$f_0$  is determined from the first boundary condition which gives

$$f_0 = \frac{2\pi \exp(R/L)}{\int_0^{2\pi} \exp(R \cos x/L) dx} \quad (28)$$

or

$$f_0 = \exp(R/L) / J_0(iR/L). \quad (29)$$

For large values of  $R/L$  the asymptotic form of  $J_0(iR/L)$  gives

$$f_0 \cong \sqrt{2\pi R/L} \quad (30)$$

Now with  $D = 5 \times 10^5 \text{ cm}^2/\text{sec}$ ,  $\lambda = 10^6 \text{ cm}$  and  $V_0 = 5 \times 10^3 \text{ cm/sec}$  it turns out that  $f_0$  is  $10^2$ . Thus to build up an ion density peak of  $3 \times 10^5 \text{ per cm}^3$  would require an average ion density of  $3 \times 10^3 \text{ per cm}^3$  in this case.

The point of half maximum density would be reached where

$$1 - \cos x = L/R \quad (31)$$

or

$$x \cong \sqrt{2L/R} \quad (32)$$

and

$$z = \lambda \sqrt{2L/R} / 2\pi \quad (33)$$

In the example just considered this would be at  $5 \times 10^{-3} \lambda$  or only 50 meters.

The ratio  $R/L$  in this example is about  $2 \times 10^3$  and  $L^{-1}$  is  $5 \times 10^4 / \tau$ . Hence the approximation of negligible creation and loss is valid so long as  $\tau$  is 500 seconds or greater. Thus at  $z = 0$  and  $z = \lambda/2$  the transport processes are creating ionization at the rate

$$2\pi V_0 N^+ / \lambda \quad (34)$$

while they are being removed chemically at the rate

$$N^+/\tau, \quad (35)$$

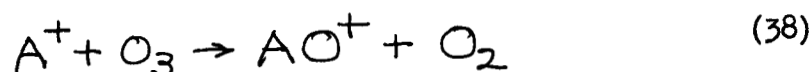
The ratio of the transport rate to the chemical rate is

$$2\pi V_0 \tau / \lambda = R \tau, \quad (36)$$

which is, in the example selected

$$3 \times 10^{-2} \tau. \quad (37)$$

At the 100 km level there are not many reactions available to ions such as  $\text{Ca}^+$ ,  $\text{Mg}^+$  and  $\text{Na}^+$ . In the case of the first two ions the reaction



should be very fast. The ozone density should be between  $10^6$  and  $10^7$  per  $\text{cm}^3$ . If a rate coefficient of the order of  $10^{-9} \text{ cm}^3/\text{sec}$  is attained by such reactions the lifetime of  $\text{Ca}^+$  or  $\text{Mg}^+$  should be at least 100 seconds. This is certainly a lower limit at 100 km. For mutual neutralization with a rate coefficient of  $10^{-6} \text{ cm}^3/\text{sec}$  the negative ion density would need to be  $10^4$  or larger to reduce the ionic lifetime below 100 seconds. Thus it would appear that the approximation (23) should be a good one above 100 km.

Where the ionic lifetime is too short the full differential equation (19) must be solved to obtain the distribution  $f(x)$ . No simple method of obtaining a solution of this inhomogeneous differential equation seems available. By means of the change of variable

$$f(x) = \left( \exp - \int_0^x R \sin u / 2L du \right) y(x) \quad (39)$$

or

$$f(x) = \left( \exp[-R(1-\cos x)/2L] \right) y(x) \quad (40)$$

it is possible to reduce the differential equation to the form

$$-\frac{1}{L} \exp[R(1-\cos x)/2L] = \frac{d^2 y}{dx^2} + \left[ \frac{R}{2L} \cos x - \left( \frac{R}{2L} \sin x \right)^2 - \frac{1}{L} \right] y \quad (41)$$

or

$$\frac{d^2 y}{dx^2} - [\alpha^2 - \beta^2 \cos 2x - \gamma \cos x] y = \omega_0^2 \exp \gamma(1-\cos x), \quad (42)$$

where

$$\omega_0^2 = 1/L \quad (43)$$

$$\beta^2 = (R\omega_0^2/2)^2/2 \quad (44)$$

$$\gamma^2 = 2\beta^2 \quad (45)$$



and

$$\alpha^2 = \omega_0^2 + \beta^2. \quad (46)$$

If  $f(x)$  is to be an even, periodic, non negative function of  $x$  so must  $y(x)$  and it can be represented by a Fourier series

$$y(x) = \sum_{n=0}^{\infty} c_n \cos nx. \quad (47)$$

If this series is inserted in the differential equation (41) and use is made of the expansion

$$\exp[-\gamma \cos x] = J_0(i\gamma) + 2 \sum_{n=1}^{\infty} i^n J_n(i\gamma) \cos nx \quad (48)$$

the following recursion relationship may be obtained for the coefficients

$c_n$

$$c_{n+2} = \frac{\gamma}{\beta^2} c_{n+1} - 2 \frac{n^2 + \alpha^2}{\beta^2} c_n + \frac{\gamma}{\beta^2} c_{n-1} + c_{n-2} = \frac{\gamma}{\beta^2} \omega_0^2 e^{\gamma} i^n J_n(i\gamma), \quad (49)$$

where

$$c_2 = 2 \frac{\alpha^2}{\beta^2} c_0 - \frac{\gamma}{\beta^2} - 2 \frac{\omega_0^2}{\beta^2} e^{\gamma} J_0(i\gamma) \quad (50)$$

and

$$c_3 = \frac{2}{\beta^2} (1 + \alpha^2) c_1 - \frac{\gamma}{\beta^2} c_2 - 4 \frac{\omega_0^2}{\beta^2} e^{\gamma} i J_1(i\gamma) \quad (51)$$

The two arbitrary constants  $c_0$  and  $c_1$  are determined by the boundary conditions (24) and (25). In the present case these may be written as follows

$$\sum_{n=0}^{\infty} c_n \int_0^{2\pi} e^{-\gamma(1-\cos x)} \cos nx \, dx = 2\pi \quad (52)$$

or

$$\sum_{n=0}^{\infty} c_n (-i)^n J_n(i\gamma) = 2\pi e^{\gamma} \quad (53)$$

and

$$\sum_{n=0}^{\infty} c_n \cos nx \geq 0 \quad (54)$$

for all  $x$ .

The ratio test for successive coefficients from the recursion formula (49) indicates that

$$\left| \frac{c_{n+1}}{c_n} \right| \sim \frac{\beta}{n} \quad \text{as } n \rightarrow \infty \quad (55)$$

and hence the series solution is rapidly convergent.

Numerical evaluation of  $f(x)$  from the series solution for  $y(x)$  would be interesting from the point of view of determining the effect of increasing the value of  $\tau^{-1}$  compared to  $2\pi V_0/\lambda$  and  $4\pi^2 D/\lambda^2$ . Since it is very likely that above 100 km  $\tau$  is long enough to justify disregard

of ion creation and loss compared to the transport processes no attempt has yet been made to obtain these solutions.

#### 4.2 THE TIME DEPENDENT EQUATION

If it is supposed that at  $t = 0$  when the velocity field is suddenly turned on the ion density is ~~un~~iform with

$$N^+ = N_0^+ \quad (60)$$

everywhere the change of ion density with time is governed by the time dependent equation

$$\tau \frac{\partial q}{\partial t} = L \frac{\partial^2 q}{\partial x^2} + R \frac{\partial}{\partial x} q \sin x - q + 1 \quad (61)$$

where  $q(x, t) = N^+ / N_0^+ \quad (62)$

is subject to the conditions

$$q \equiv 1 \quad (63)$$

everywhere at  $t = 0$  and, as before

$$\int_0^{2\pi} q(x, t) dx = 2\pi \quad (64)$$

$$q(x, t) \geq 0 \quad (65)$$

at all times. If a steady state exists then

$$q(x, t) \rightarrow f(x) \quad \text{as } t \rightarrow \infty. \quad (66)$$

where  $f(x)$  is the steady state solution of the steady state equation already discussed. It is obvious that the simple separable solution

$$g(x, t) = f(x) T(t) \quad (67)$$

cannot satisfy these conditions since  $g = 1$  at  $t = 0$  would imply the contradiction

$$T(t) = f(x)^{-1} \quad (68)$$

Hence, if separable solutions exist, it is necessary to combine two linearly independent solutions to form

$$g(x, t) = (f_1 T_1 + f_2 T_2) / 2 \quad (69)$$

such that

$$(f_1 T_1 + f_2 T_2) / 2 = 1 \quad (70)$$

at  $t = 0$  and such that

$$\frac{\partial g}{\partial t} = \frac{1}{2} f_1 \frac{dT_1}{dt} + \frac{1}{2} f_2 \frac{dT_2}{dt} \rightarrow 0 \text{ as } t \rightarrow \infty \quad (71)$$

if that is possible

Separable solutions  $f_i T_i$  exist if

$$\frac{1}{T_i} \frac{1}{L} \frac{\partial T_i}{\partial t} - \frac{1}{L f_i T_i} = F_i(x) \equiv \frac{1}{f_i} \frac{\partial^2 f_i}{\partial x^2} + \frac{R}{L f_i} \frac{df_i}{dx} - \frac{1}{L} \quad (72)$$

The functions  $T_i(t)$  must therefore be solutions of the differential equation

$$\frac{dT_i}{dt} = \frac{F_i L}{\tau} T_i + \frac{1}{f_i \tau} \quad (73)$$

where the dependence on  $x$  of the two terms in the right hand must balance each other. The solution of this differential equation is

$$T_i(t) = \left( T_{i0} + \frac{1}{F_i L f_i} \right) e^{\frac{F_i L}{\tau} t} - \frac{1}{F_i L f_i} \quad (74)$$

where  $F_i < 0$  and

$$T_i \rightarrow T_0 \text{ as } t \rightarrow 0 \quad (75)$$

$$T_i \rightarrow -\frac{1}{F_i L f_i} \text{ as } t \rightarrow \infty. \quad (76)$$

Thus as  $t \rightarrow \infty$

$$F_i(x) f_i(x) \rightarrow \frac{1}{L} \quad (77)$$

as it should.

The characteristic time of relaxation for the distribution is thus

$$\frac{\tau}{FL} = \frac{\tau}{\frac{L}{f} \frac{d^2 f}{dx^2} + \frac{R}{f} \frac{d}{dx} f \sin x - 1} \quad (78)$$

This is of the order of

$$\tau, \quad \lambda / 2\pi V_0 \quad \text{or} \quad \lambda^2 / 4\pi^2 D \quad (79)$$

whichever is smallest. The time is typically on the order of a few minutes.

#### 4.3 Steady State: Removal by Recombination

If the ionic species whose density is  $N_i$  is removed primarily by direct recombination with electrons the volume rate of loss is given by

$$\alpha_i N_i^+ N_e \quad (80)$$

where, however,

$$N_i^+ \neq N_e \quad (81)$$

necessarily. In this case, too, if  $\alpha_i$  is very much less than the recombination coefficients for the major ions a large density of the minor species can build up at the nodes in the east-west wind pattern.

In this case the steady state condition parallel to the one of Whitehead (7, 8) in which diffusion is neglected becomes

$$\alpha N_{eo} R \frac{d}{dx} N_i^+ \sin x - \alpha_i N_i^+ N_e + \alpha_i N_{io}^+ N_{eo} = 0 \quad (82)$$

where

$$N_{io}/N_{eo} = \alpha \ll 1 \quad (83)$$

This equation may be written in terms of  $f(x)$

$$R \frac{d}{dx} f \sin x - f \frac{N_e}{N_{eo}} + 1 = 0 \quad (84)$$

The maximum value of  $f$  will occur at  $x=0$  and there its value will be given by

$$1 - f_M \left( \frac{N_e}{N_{eo}} \right)_M + R f_M = 0 \quad (85)$$

If the maximum in  $f$  is to be so large that

$$N_i^+ \cong N_e \quad (86)$$

at  $x=0$  then the condition (85) becomes

$$1 - r f_M^2 + R f_M = 0 \quad (87)$$

or

$$f_M = (R + \sqrt{R^2 + 4r}) / 2r \quad (88)$$

if  $R \gg 1$  this condition may be written as

$$f_M = R / r, \quad (89)$$

Since  $r$  is the relative average abundance of the minor ionic species it is clear that unless  $r$  is too small the minor ion abundance at the maxima can build up to densities comparable to those attained by major species with the same recombination coefficient. Of course, if  $r$  is too small the approximation that  $N_i$  and  $N_e$  are equal at the peak cannot hold. The attractive feature of this model is that  $\alpha_{\text{eff}}$  for the major ions can be allowed to be of the order of  $5 \times 10^{-7} \text{ cm}^3/\text{sec}$  inhibiting any build up in their density while in principle the minor ion recombination coefficient might be as small as  $3 \times 10^{-12} \text{ cm}^3/\text{sec}$ . Thus  $R$  could be very large indeed if the minor ion had to wait around for an electron to neutralize it. It is much more likely however that some other mechanism will lead to neutralization more effectively than radiative recombination. Thus the treatment in sections 4.1 is more likely to be applicable.



The minimum value of  $f$  occurs at  $x = \pi$  and is given by

$$f_m = (R - \sqrt{R^2 + 4n}) / 2n \approx R^{-1} \quad (90)$$

The rest of the Whitehead treatment can be modified appropriately to accommodate this model by appropriate insertions of the parameter  $r$ .

## 5. DISCUSSION AND CONCLUSION

As support for this mechanism of Sporadic E formation it is possible to cite the appearance of metallic ions in the mass spectra obtained in the lower E regions<sup>(17, 18)</sup>. The abundances of these ions appears to be adequate if the calibration of the mass spectrometers is correct. Average densities of the order of  $10^3$  are required between 95 and 120 km.

It is also suggestive that the frequency of occurrence of Sporadic-E ( $fE_S > 5 M_C$ ) varies during the year in temperate latitudes in a way not unlike the frequency of radar meteor echoes<sup>(11, 12)</sup>. This same temporal variation holds for the abundance of sodium atoms in the daytime as deduced from dayglow observations of the sodium D lines<sup>(2, 19)</sup>. It would be most interesting of course to identify mass spectrometrically the ions which populate the Sporadic-E layer. However, lacking such evidence, there does not appear to be any reason not to postulate that these ions are metallic and that the Sporadic-E layer is a permanent feature at the E-W wind nodes. It becomes observable from the ground only when the rate of influx of meteoric material suffers an upward perturbation and temporarily enhances the supply of the requisite ions.

The fact that the daytime steady state ionosphere requires that  $O_2^+$  and  $NO^+$  recombine at rates which demand dissociative recombination coefficients of the order of  $1-5 \times 10^{-7} \text{ cm}^3/\text{sec}$  and the fact that these values are also given by laboratory measurements would suggest that models for time dependent ionospheric phenomena avoid contradiction with this result. The present model, while not demonstrably representing the real situation, does have this merit. It shows that the wind shear mechanism for Sporadic-E does not necessarily entail a contradiction with the laboratory and steady state results for the recombination coefficients of  $O_2^+$  and  $NO^+$ .

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